DEFINITIONS

- A grammar is said to be **left-recursive** if it has a non-terminal A s.t. there is a derivation $A \Rightarrow A\alpha$ for some string α of terminals and non-terminals.
- **Immediate** or **simple** left-recursion is when the grammar contains a production of the form $A \rightarrow A\alpha$ for some string α of terminals and non-terminals.

ELIMINATING LEFT RECURSION

- Eliminating simple left-recursion:
  - $S \rightarrow S\alpha \mid \beta$ where $\alpha, \beta \in (N \cup T)^*$ but don't start with S
  - Language generated is $\beta\alpha \ldots \alpha$ (0 or more α's) = $\beta\alpha^*$
  - Replace by
    $S \rightarrow \beta S'$
    $S' \rightarrow \alpha S' \mid \varepsilon$
- More complex situation:
  - $S \rightarrow S\alpha_1 \mid \ldots \mid S\alpha_n \mid \beta_1 \mid \ldots \mid \beta_m$
    where α's, β's $\in (N \cup T)^*$ but don't start with S
  - Replace by:
    $S \rightarrow BS'$
    $B \rightarrow \beta_1 \mid \ldots \mid \beta_m$
    $S' \rightarrow \varepsilon \mid \alpha_1 S' \mid \ldots \mid \alpha_n S'$
- Eliminating general left recursion:
  - Make a list of all non-terminals in an order
  - For each non-terminal
    - Examine all its productions. If the right-hand side begins with a non-terminal earlier in the list (e.g. $B \rightarrow A\beta$)
      1. Try to reorder B before A if it solves the problem,
      2. Otherwise look at the earlier productions.
         - If these are: $A \rightarrow \alpha_1 \mid \ldots \mid \alpha_n$
         - Substitute $B \rightarrow \alpha_1\beta \mid \ldots \mid \alpha_n\beta$
         - Continue until no right-hand side begins with a non-terminal earlier in the list.
    - Remove the immediate left recursions (if any) for this non-terminal